

# Multiplication and

3

*Here Comes the Band!*

## Learning Goals

- use different mental math strategies to multiply and divide
- multiply by 0, 1, and 10
- divide by 1
- recall multiplication and division facts
- identify and describe patterns in a multiplication chart
- relate multiplication and division
- pose and solve story problems using multiplication and division
- write and solve equations



# Division Facts

## Key Words

array

factor

product

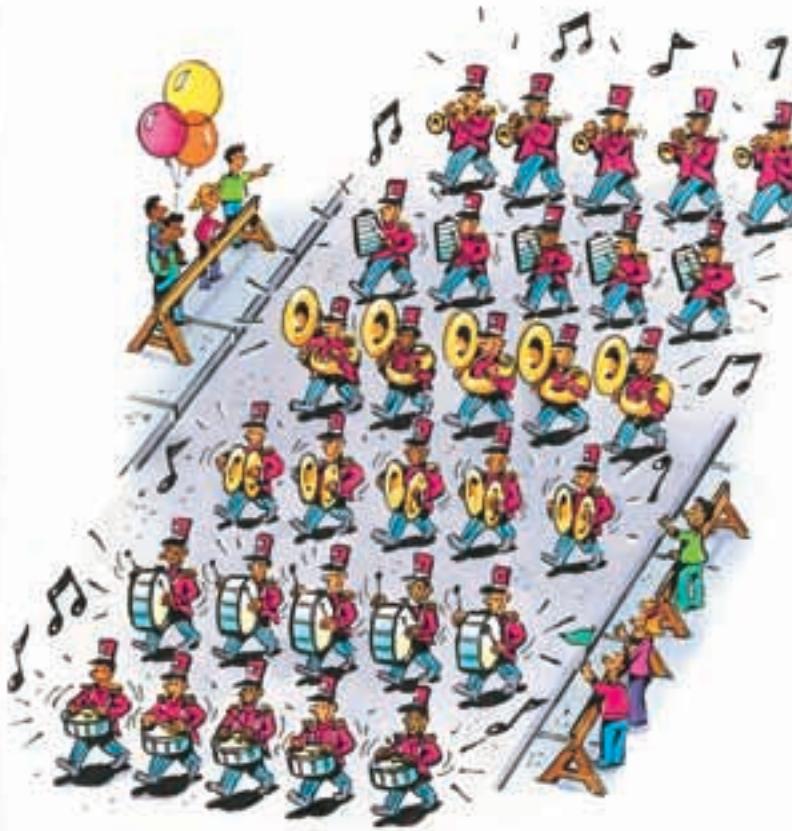
row

column

multiples

related facts

In a parade, there are many bands.



- How many people are in this band?
- How did you find out?
- How else can you find out how many people are in the band?
- How many different ways can you find out how many are in the band?



# Using Doubles to Multiply

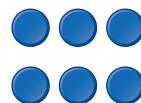
An **array** shows objects arranged in equal rows.

You can use an array to multiply.

To find  $2 \times 3$ , make an array of 2 rows of 3.

2 rows of 3 counters

$$2 \times 3 = 6$$



This is a multiplication fact.

## Explore



You will need 1-cm grid paper and scissors.

► Use the grid paper.

Draw an array for  $3 \times 4$ .

Cut out the array.

Record a multiplication fact for the array.

► Use your array and your partner's array to make a larger array.

Record a multiplication fact for this new array.

Arrange your arrays another way.

Record another multiplication fact.

► With your partner, choose one of these:

$$2 \times 4 \quad 3 \times 5 \quad 4 \times 6$$

Cut out an array to show the fact.

Record a multiplication fact for the array.

Put your array together with your partner's array.

Record a multiplication fact for this new array.

Arrange the arrays another way.

Record a multiplication fact for that array.

## Show and Share

Show your arrays and multiplication facts to another pair of students. What do you notice when you double the size of an array?

### Connect

In a multiplication fact, you multiply **factors** to get a **product**.

$$\begin{array}{r} 4 \\ \times \\ 5 \\ \hline \text{factor} \quad \text{factor} \quad \text{product} \end{array}$$

When you double a number, you are multiplying by 2.

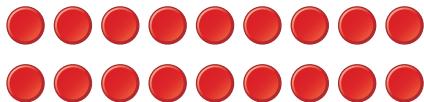
Doubling is a strategy you can use to multiply.

Here are three ways you can use doubling to multiply.

- Use doubling to multiply by 4.

To find  $4 \times 9$ :

First find  $2 \times 9$ , then double.



$$2 \times 9 = 18$$



$$2 \times 9 = 18$$

$$18 + 18 = 36$$

$$4 \times 9 = 36$$

- Use repeated doubling to multiply by 8.

To find  $8 \times 8$ :

First think of  $2 \times 8 = 16$ , then double, and then double again.



$$2 \times 8 \text{ is } 8 + 8 = 16$$

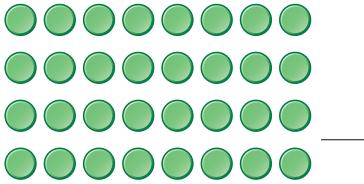
$$\text{So, } 2 \times 8 = 16$$



$$4 \times 8 \text{ is double } 2 \times 8$$

$$16 + 16 = 32$$

$$\text{So, } 4 \times 8 = 32$$



$$8 \times 8 \text{ is double } 4 \times 8$$

$$32 + 32 = 64$$

$$\text{So, } 8 \times 8 = 64$$

► Begin with a fact you know.  
Double one of the factors, then multiply.

You know  $2 \times 3 = 6$ .

You can double the factor 2 to get 4.

$$4 \times 3 = 12$$

Or, you can double the factor 3 to get 6.

$$2 \times 6 = 12$$

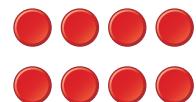
When you double a factor, the product doubles.



## Practice

Use counters when they help.

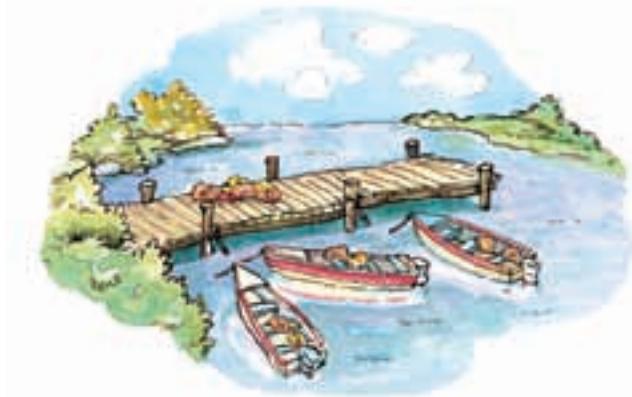
1. a) Write a multiplication fact for this array.  
b) Double one factor in the multiplication fact.  
Make an array for this new fact.  
Write a multiplication fact.



2. Six students can travel in 1 boat.

- How many students can travel in 2 boats?
- How many students can travel in 4 boats?
- How many students can travel in 8 boats?

How do you know?



3. Multiply. What strategies did you use?

a) $2 \times 6$	b) $4 \times 6$	c) $8 \times 6$
d) $8 \times 2$	e) $8 \times 4$	f) $8 \times 8$

4. Write two facts that help you find the product of  $6 \times 8$ .

5. Find the product. What strategies did you use?

a) $2 \times 7$	b) $4 \times 7$	c) $8 \times 7$
d) $2 \times 5$	e) $4 \times 5$	f) $8 \times 5$

## Math Link

### Measurement

To find how many cents are in 8 nickels, multiply:  $8 \times 5$



6. How can you use  $3 \times 6$  to find  $6 \times 6$ ?

Show your work.

7. Find the product. What strategies did you use?

a)  $7 \times 4$

b)  $6 \times 4$

c)  $7 \times 6$

d)  $3 \times 4$

8. There are bicycles and wagons in the playground.

Each bicycle has 2 wheels.

Each wagon has 4 wheels.

Eli counted 28 wheels.

How many bicycles and wagons might there be?

How many different ways can you find the answer?

Use multiplication facts to show your work.



9. Write a multiplication story problem about wheels.

Solve your problem. Show your work.

10. What might each missing number be?

How many answers can you find?

a)  $\square \times \triangle = 16$

b)  $\circ \times \square = 24$

### Reflect

How do you know that doubling the product of  $2 \times 6$  is the same as finding  $4 \times 6$ ?

Use words, numbers, or pictures to explain.

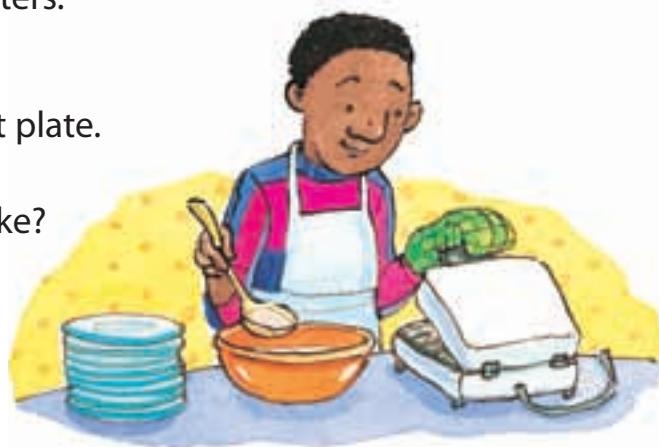
## Multiplying by 1, by 0, and by 10

## Explore



You will need paper plates and counters.

- Mark makes waffles for his family.  
He puts each waffle on a different plate.  
Mark uses 5 plates.  
How many waffles does Mark make?  
How can you use a multiplication  
fact to show this?
  
- Mark has 3 empty plates.  
How many waffles are on  
these plates?  
How can you use a multiplication  
fact to show this?
  
- Mark has 6 trays.  
He stacks 10 waffles on each tray.  
How many waffles does Mark stack?  
How can you use a multiplication  
fact to show this?

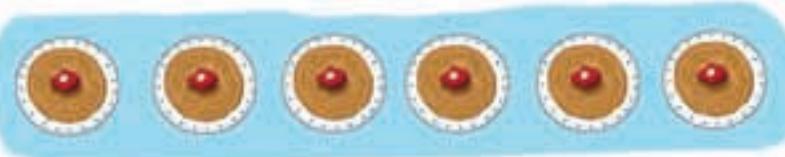


## Show and Share

Show your work to another pair of classmates.  
What is special about multiplying by 1?  
What is special about multiplying by 0?  
What is special about multiplying by 10?

## Connect

► Marie made French crêpes for her friends.



She put 1 crêpe on each plate.

Marie used 6 plates.

How many crêpes did Marie make?

Think: 6 groups of 1 is  $6 \times 1$ .

$$\begin{array}{r} 6 \\ \text{plates} \end{array} \times \begin{array}{r} 1 \\ \text{crêpe} \end{array} = \begin{array}{r} 6 \\ \text{crêpes} \\ \text{in all} \end{array}$$

Also,  $1 \times 6 = 6$

► Marie had 5 empty plates.



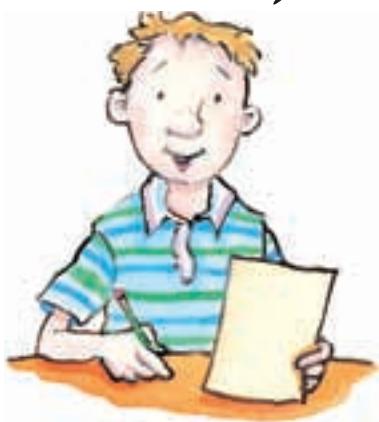
How many crêpes are on these plates?

Think: 5 groups of 0 is  $5 \times 0$ .

$$\begin{array}{r} 5 \\ \text{plates} \end{array} \times \begin{array}{r} 0 \\ \text{crêpes} \end{array} = \begin{array}{r} 0 \\ \text{crêpes} \\ \text{in all} \end{array}$$

Also,  $0 \times 5 = 0$

When 1 is a factor, the product is always the other factor.



When 0 is a factor, the product is always 0.



► Marie made many crêpes and served them on trays.



She put 10 crêpes on each tray.

Marie used 3 trays.

How many crêpes did Marie serve?

Think: 3 groups of 10 is  $3 \times 10$ .

$$\begin{array}{rcl} 3 & \times & 10 \\ \text{trays} & & \text{crêpes} \\ & & \text{crêpes} \\ & & \text{in all} \end{array}$$

Also,  $10 \times 3 = 30$

## Practice

1. Multiply.

a)  $0 \times 6$       b)  $1 \times 5$       c)  $0 \times 7$   
d)  $1 \times 3$       e)  $10 \times 8$       f)  $10 \times 3$

2. Jessica buys 6 single-scoop ice cream cones.

How many scoops of ice cream does Jessica buy?

Show your answer using pictures and numbers.



3. Mario has 3 empty ice cream cones.

How many scoops of ice cream does Mario have?

Write a multiplication fact.

4. Solve each equation.

a)  $6 \times \square = 6$       b)  $\square \times 3 = 0$   
c)  $\square \times 5 = 5$       d)  $2 \times \square = 0$   
e)  $\square \times 10 = 90$       f)  $10 \times \square = 70$

5. Write a story problem that can be solved by multiplying by 0.

Write an equation for the problem.

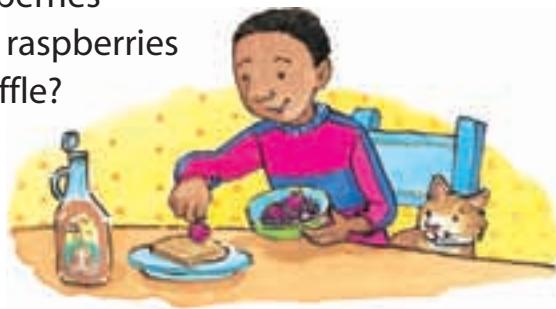
Give your problem to a classmate to solve.



6. Mark puts these fruits on his waffle:

- one strawberry
- twice as many raspberries as strawberries
- three times as many blueberries as raspberries

How many raspberries are on the waffle?  
How many blueberries?  
How do you know?



7. a) Is it easier to solve  $24 \times 1$  or  $24 \times 2$ ? Explain.

b) Is it easier to solve  $6 \times 0$  or  $66 \times 0$ ? Explain.

8. You have 6 dimes.

Your brother has 9 nickels.

Who has more money?

How do you know?

Show your work.



9. Which is greater:

a)  $1 \times 4$  or  $0 \times 9$ ?   b)  $10 \times 2$  or  $9 \times 1$ ?   c)  $0 \times 8$  or  $3 \times 1$ ?

How do you know?

10. a) Explain why the answer is always 0 when you multiply by 0.

b) Explain why the answer is always the other number when you multiply by 1.

### Reflect

What can you say about:

- multiplying by 0?
- multiplying by 1?
- multiplying by 10?

Use words, pictures, or numbers to explain.



# Using Skip Counting to Multiply

## Explore



You will need copies of these charts.

**Hundred Chart**

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

**Multiplication Chart**

x	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9
2	2	4	6	8	10	12	14	16	18
3	3	6							
4	4	8							
5	5	10							
6	6	12							
7	7	14							
8	8	16							
9	9	18							

► Use the hundred chart.

Start at 3 and count on by 3s.

Use the numbers you count to fill in the next row and column in the multiplication chart.

► Repeat the activity.

Start at 4 and count on by 4s.

Start at 5 and count on by 5s.

Count on by other numbers until you have filled in the chart.

What patterns do you see in the multiplication chart?

## Show and Share

Share the patterns you found with another pair of students.

What patterns did you find in the ones digits? The tens digits?

## Connect

Here are some ways to use skip counting to multiply mentally.

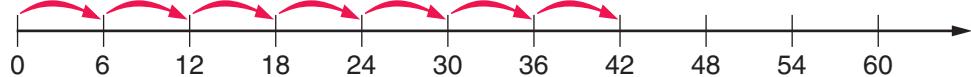
► Use skip counting patterns.

For example,

To find  $7 \times 6$ , skip count by 6 seven times:

6, 12, 18, 24, 30, 36, 42

These numbers are multiples of 6.



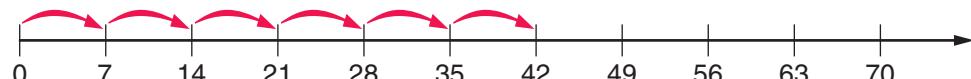
7 steps of 6 is 42.

$$7 \times 6 = 42$$

Another way is to skip count by 7 six times:

7, 14, 21, 28, 35, 42

These numbers are multiples of 7.



6 steps of 7 is 42.

$$7 \times 6 = 42$$

► Skip count from a known fact.

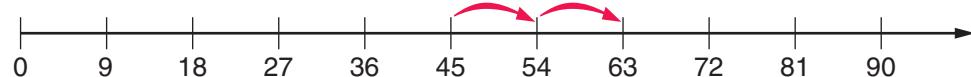
You know  $9 \times 5 = 45$ .

To find  $9 \times 6$ :

Skip count by 9 to add one more group of 9.

“45... then 54”

$$\text{So, } 9 \times 6 = 54$$



To find  $9 \times 7$ :

You know  $9 \times 5 = 45$ .

Skip count by 9 to add two more groups of 9.

“45... then 54... then 63”

$$\text{So, } 9 \times 7 = 63$$

## Practice

1. Use a hundred chart
  - a) Start at 4. Skip count by 4 to find the multiples of 4.  
List them.
  - b) Start at 8. Skip count by 8 to find the multiples of 8.  
List them.
  - c) Compare the numbers in both lists.  
What patterns do you see?
2. Skip count to find the missing numbers.
  - a) 3, 6, 9, \_\_, \_\_, \_\_, \_\_
  - b) 5, 10, 15, \_\_, \_\_, \_\_, \_\_
  - c) 6, 12, 18, \_\_, \_\_, \_\_, \_\_
  - d) 7, 14, 21, \_\_, \_\_, \_\_, \_\_
3. Use skip counting.  
Copy and complete these rows from a multiplication chart.

x	3	4	5	6	7	8	9
3	9	12					
4	12	16					

Look at the completed rows for  $\times 3$  and  $\times 4$ . How are they related?

4. Multiply.  
How does knowing the first fact help you complete the second fact?
  - a)  $6 \times 5 = 30$ , then  $7 \times 5 = \square$
  - b)  $3 \times 6 = 18$ , then  $4 \times 6 = \square$
  - c)  $5 \times 8 = 40$ , then  $6 \times 8 = \square$
5. There are 8 pairs of students ready to learn a French dance.  
How many students are there?  
Show your work.
6. Nicole collects nickels.  
Could she have 45 cents?  
Show your work.
7. Jillian knows that  $3 \times 7 = 21$ .  
How can she use that fact to find  $5 \times 7$ ?



8. Play this game with a partner.

You will need a number cube labelled 4, 5, 6, 7, 8, 9; a counter; and a hundred chart.

Choose a target number between 50 and 100.

Put the counter on that number.

Roll the number cube to find the start number.

Use the same number to count on.

Have your partner say if you will “hit” the target when you count on.

Count on to check.

Or, explain how you know.

Trade roles.

Take turns to roll and check.



9. Here is part of a multiplication chart.

Each shape in the chart represents a number.

a) Which number does each shape represent?

b) What strategies did you use to find each number?

c) How are the rows for  $\times 5$  and  $\times 6$  related?

Show your work.

$\times$	5	6	7	8	9
5	25	30	★	40	45
6	30	□	42	48	○
7	35	42	49	△	63
8	●	48	56	64	72
9	45	54	○	72	□



10. Use a calculator to count on.

Press the keys.

Record what you see on the screen.

Write about the patterns you see.

a) Press:  $\text{ON/C}$   $2$   $+$   $=$   $=$   $=$   $=$   $=$   $=$   $=$   $=$

b) Press:  $\text{ON/C}$   $5$   $+$   $=$   $=$   $=$   $=$   $=$   $=$   $=$   $=$

c) Press:  $\text{ON/C}$   $9$   $+$   $=$   $=$   $=$   $=$   $=$   $=$   $=$

Reflect

How does skip counting help you multiply?

Use words and numbers to explain.

# Other Strategies for Multiplying

## Explore



Play this game with a partner.

### Cross-Out Product

You will need a number cube labelled 4, 5, 6, 7, 8, 9; and 2 copies of this game board.

4	6	9	12	14
18	20	24	27	32
35	40	42	45	49
56	63	64	72	81

- One person rolls the number cube to get a factor.
- Each person thinks of a multiplication fact that uses that factor.
- Record the fact you use.
- Cross out the product on your game board.



- Take turns to roll the number cube.
- The first person to cross out all the products is the winner.

### Show and Share

Talk with your partner about the strategies you used to multiply. What mental math strategies did you use?

## Connect

Another way to multiply is to add groups to facts you know.

► Use facts with 2 to multiply by 3.

First, multiply by 2.

Then, add another group.

To find  $3 \times 7$ :

Think:  $2 \times 7 = 14$

$1 \times 7 = 7$

$14 + 7 = 21$

The diagram shows a thought bubble with the text "Think:  $2 \times 7 = 14$ ". A red arrow points from this text to the number 14. Below it is the text " $1 \times 7 = 7$ ". To the right of these two lines is a large right-pointing arrow. To the right of the arrow is the equation  $14 + 7 = 21$ .

So,  $3 \times 7 = 21$

► Use facts with 5 to multiply by 6.

First, multiply by 5.

Then, add another group.

To find  $6 \times 7$ :

Think:  $5 \times 7 = 35$

$1 \times 7 = 7$

$35 + 7 = 42$

The diagram shows a thought bubble with the text "Think:  $5 \times 7 = 35$ ". A red arrow points from this text to the number 35. Below it is the text " $1 \times 7 = 7$ ". To the right of these two lines is a large right-pointing arrow. To the right of the arrow is the equation  $35 + 7 = 42$ .

So,  $6 \times 7 = 42$

► Use facts with 5 and 2 to multiply by 7.

Break the 7 into a fact with 5 and a fact with 2.

To find  $7 \times 8$ :

Think:  $5 \times 8 = 40$

$2 \times 8 = 16$

$40 + 16 = 56$

The diagram shows a thought bubble with the text "Think:  $5 \times 8 = 40$ ". A red arrow points from this text to the number 40. Below it is the text " $2 \times 8 = 16$ ". To the right of these two lines is a large right-pointing arrow. To the right of the arrow is the equation  $40 + 16 = 56$ .

So,  $7 \times 8 = 56$

► Use facts with 10 to multiply by 9.

To find  $9 \times 4$ , think:

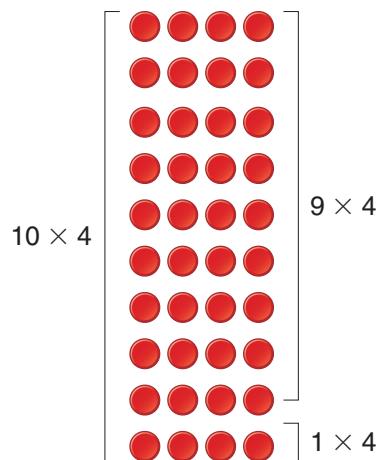
$$10 \times 4 \text{ less } 1 \times 4$$

$$10 \times 4 = 40$$

Subtract one group of 4.

$$40 - 4 = 36$$

$$9 \times 4 = 36$$



When 9 is a factor,  
I think about multiplying  
by 10!

► Use a half then double, to multiply by an even factor. Here is another way to find  $6 \times 7$ :

Choose the

even factor, 6.

Half of 6 is 3.

Think of  $3 \times 7$ , then double.



$$3 \times 7 = 21$$



$$3 \times 7 = 21$$



$$3 \times 7 = 21$$



$$21 + 21 = 42$$

$$\text{So, } 6 \times 7 = 42$$

When I multiply by an even number, I think about how I can use doubles.

## Practice

1. Multiply. What strategies did you use?

a)  $3 \times 6$

b)  $6 \times 3$

c)  $7 \times 5$

d)  $5 \times 7$

e)  $3 \times 3$

f)  $6 \times 6$

g)  $7 \times 7$

h)  $7 \times 6$

2. Multiply. What strategies did you use?

a)  $9 \times 2$

b)  $9 \times 4$

c)  $9 \times 8$

d)  $9 \times 5$

e)  $9 \times 3$

f)  $9 \times 6$

g)  $9 \times 9$

h)  $9 \times 7$

3. Name two facts to help you find each product.

a)  $7 \times 7$

b)  $3 \times 8$

c)  $6 \times 6$

d)  $8 \times 6$

4. Alexis is having a birthday party in 4 weeks.

How many days does she have to wait?

5. You have 6 nickels and 4 dimes.  
How many cents do you have?  
Draw a picture.



6. How can you find the product of  $9 \times 7$ ,  
if you know the product of  $9 \times 5$ ?  
Show your work.

7. Rabia bought 4 bags of oranges.  
Each bag had 6 oranges.  
How many oranges did Rabia buy?



8. Write a story problem using the information given below.  
Write an equation for the problem.  
Solve the problem.  
Include a multiplication fact as part of your answer.

a) There are 4 wheels on a wagon. There are 8 wagons.  
b) There are 2 wheels on a bicycle. There are 9 bicycles.  
c) There are 3 wheels on a tricycle. There are 7 tricycles.

9. Write a story problem for this question:  $6 \times 7$   
Solve your problem. Show your work.

10. Without multiplying, how do you know that the product of  $6 \times 5$  is less than the product of  $7 \times 6$ ?

11. Write a story problem for this equation:  $7 \times 8 = \square$   
Solve the equation to solve the problem.

### Reflect

You have learned different ways to multiply.  
Which way do you prefer?  
Why?

### At Home



Ask family members about their strategies for remembering multiplication facts.  
How do their strategies compare with yours?

# Using Patterns in a Multiplication Chart



The **row** and **column** for the same factor have the same numbers.

The factors 6 and 7 are highlighted.

They show that:

$$6 \times 7 = 42 \text{ and } 7 \times 6 = 42$$

Column

Row

x	1	2	3	4	5	6	7
1	1	2	3	4	5	6	7
2	2	4	6	8	10	12	14
3	3	6	9	12	15	18	21
4	4	8	12	16	20	24	28
5	5	10	15	20	25	30	35
6	6	12	18	24	30	36	42
7	7	14	21	28	35	42	49

## Explore



Look at this multiplication chart. How is it the same as the chart above? How is it different? Use the chart to write ten multiplication facts.

## Show and Share

Share your facts with another pair of students.

What patterns do you see in products with factors of 2? With factors of 5?

What other patterns can you find?

x	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	10	12	14	16	18
3	0	3	6	9	12	15	18	21	24	27
4	0	4	8	12	16	20	24	28	32	36
5	0	5	10	15	20	25	30	35	40	45
6	0	6	12	18	24	30	36	42	48	54
7	0	7	14	21	28	35	42	49	56	63
8	0	8	16	24	32	40	48	56	64	72
9	0	9	18	27	36	45	54	63	72	81

## Connect

You can use patterns to remember multiplication facts.

- In a multiplication chart, there are matching numbers on each side of the diagonal from 0 to 81.

You can use these numbers to help you remember multiplication facts.

x	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	10	12	14	16	18
3	0	3	6	9	12	15	18	21	24	27
4	0	4	8	12	16	20	24	28	32	36
5	0	5	10	15	20	25	30	35	40	45
6	0	6	12	18	24	30	36	42	48	54
7	0	7	14	21	28	35	42	49	56	63
8	0	8	16	24	32	40	48	56	64	72
9	0	9	18	27	36	45	54	63	72	81

Try to remember as many facts as you can.



If you know:



$$7 \times 6 = 42$$

$$8 \times 5 = 40$$

$$9 \times 4 = 36$$

then you know:



$$6 \times 7 = 42$$

$$5 \times 8 = 40$$

$$4 \times 9 = 36$$

► You can use patterns to remember multiplication facts with 9.

The number multiplied by 9 is always 1 more than the tens digit in the product; for example:

6 is 1 more than 5.

7 is 1 more than 6.

$$\begin{aligned}1 \times 9 &= 9 \\2 \times 9 &= 18 \\3 \times 9 &= 27 \\4 \times 9 &= 36 \\5 \times 9 &= 45 \\6 \times 9 &= 54 \\7 \times 9 &= 63 \\8 \times 9 &= 72 \\9 \times 9 &= 81\end{aligned}$$

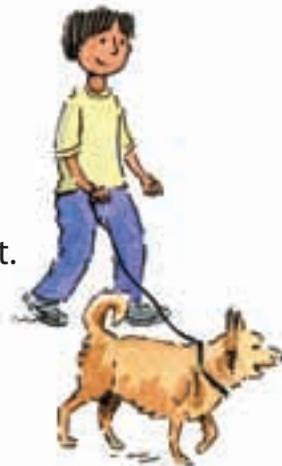
The digits in the product always add to 9; for example:

$$3 + 6 = 9$$

$$4 + 5 = 9$$

## Practice

- What are the missing numbers?  
**a)**  $7 \times 6 = \square \times 7$    **b)**  $8 \times 3 = 3 \times \square$    **c)**  $\square \times 9 = 9 \times 6$    **d)**  $4 \times \square = 6 \times 4$   
Explain how you know.
- Multiply. Which strategies did you use?  
**a)**  $5 \times 8$    **b)**  $3 \times 9$    **c)**  $4 \times 4$    **d)**  $6 \times 7$    **e)**  $2 \times 7$   
**f)**  $9 \times 5$    **g)**  $8 \times 8$    **h)**  $8 \times 6$    **i)**  $1 \times 7$    **j)**  $4 \times 7$
- How can you use patterns to find each product?  
**a)**  $7 \times 9$    **b)**  $8 \times 9$    **c)**  $9 \times 9$    **d)**  $9 \times 6$    **e)**  $5 \times 9$
- If you know  $7 \times 8$ , what else do you know?
- Show how you know each product.  
**a)**  $8 \times 5$    **b)**  $8 \times 6$    **c)**  $6 \times 9$    **d)**  $7 \times 8$    **e)**  $6 \times 3$
- How many days are there in 8 weeks? 9 weeks? How do you know?
- Write a multiplication fact for each product.  
How many different facts can you find for each product?  
**a)** 12   **b)** 16   **c)** 18   **d)** 24   **e)** 36
- Use a copy of the multiplication chart.  
Colour a design on the chart.  
Write a multiplication fact for each product you coloured.  
Exchange your facts with a classmate.  
Draw your classmate's design on another multiplication chart.
- Yana walks his dog every day for 2 hours.  
How many hours does Yana walk his dog in 5 weeks?  
Show your work.



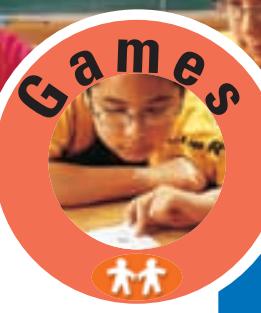
## Reflect

A student cannot remember that  $9 \times 7 = 63$ .

What strategy might the student use to remember this fact?

Use words, pictures, or numbers to explain.

# Array, Array!



You will need scissors and several sheets of 1-cm grid paper. Cut out an array for each multiplication fact from  $2 \times 2$  to  $9 \times 9$ . You should have 36 arrays.

For each array:

- Write the products of factors on one side.
- Write the product on the other side.

		8	×	2		
		2	×	8		

16
----

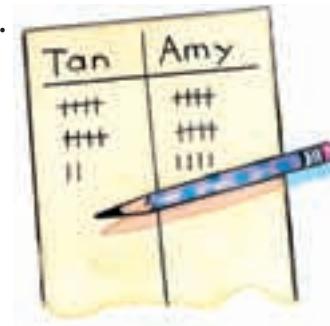
## Game 1 Matching Arrays

- Spread out the arrays.  
18 cards should have grid side up.  
18 cards should have grid side down.
- Take turns to choose an array and say what is on its other side.  
If you are correct, you keep the array.  
If you are incorrect, put the array back on the table.
- The winner is the student with more arrays at the end of the game.



## Game 2 Who Has the Greater Product?

- Deal the cards, grid side up.
- Each student places one card on the table.  
The student with the greater product takes both cards.
- Subtract the products.  
The answer is the number of points the student gets.  
Use a tally chart to keep score.
- The winner is the student with more points at the end of the game.

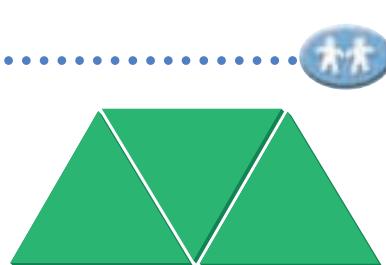


## 6

## Strategies Toolkit

## Explore

Mrs. Chan has triangular tables in her library. She arranges the tables into one long row. The tables fit together as shown. One person can sit at each side of a table. Mrs. Chan needs to seat 25 people. How many tables does she need?



## Show and Share

Describe the strategy you used to solve the problem.

## Connect

Mr. Pasma has to seat 32 people at square tables. He arranges the tables into one long row. One person can sit at each side of a table. How many tables does Mr. Pasma need?



## Strategies

- Make a table.
- Use a model.
- Draw a picture.
- Solve a simpler problem.
- Work backward.
- Guess and test.
- Make an organized list.
- Use a pattern.

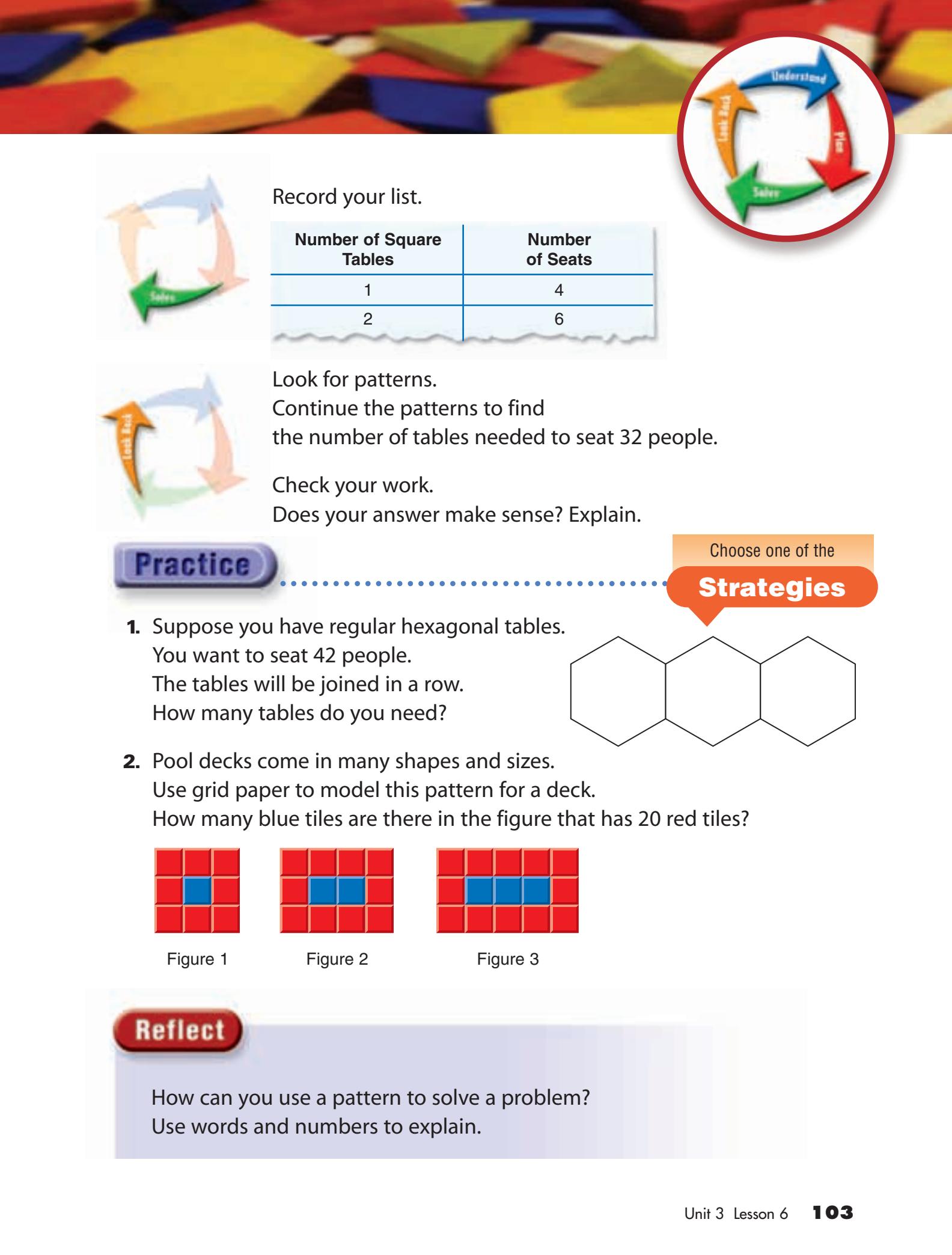


What do you know?

- The tables are square.
- There is a maximum of 4 seats at each table.
- The tables are arranged in a row.

Think of a strategy to help you solve the problem.

- You can **use a pattern**.
- Use orange Pattern Blocks to model the tables.
- List the numbers of tables and the numbers of seats.



Record your list.



Number of Square Tables	Number of Seats
1	4
2	6

Look for patterns.

Continue the patterns to find  
the number of tables needed to seat 32 people.

Check your work.

Does your answer make sense? Explain.

## Practice

Choose one of the

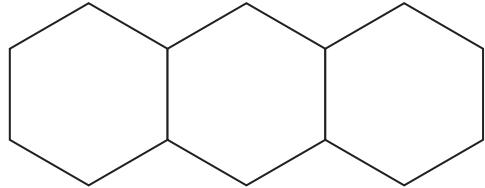
### Strategies

- Suppose you have regular hexagonal tables.

You want to seat 42 people.

The tables will be joined in a row.

How many tables do you need?



- Pool decks come in many shapes and sizes.

Use grid paper to model this pattern for a deck.

How many blue tiles are there in the figure that has 20 red tiles?

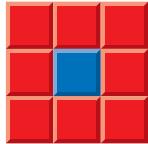


Figure 1

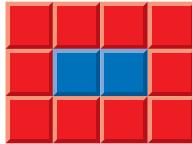


Figure 2

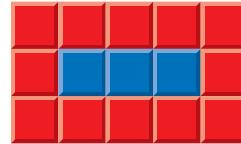


Figure 3

## Reflect

How can you use a pattern to solve a problem?

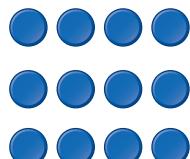
Use words and numbers to explain.

# Using Arrays to Divide

You used multiplication facts to describe an array.

3 rows of 4 counters

$$3 \times 4 = 12$$



Explain how the counters are divided into rows.

Write the division fact.

## Explore

.....



You will need counters.

There are 24 children in the parade.

They have to line up in equal rows.

How many children could be in each row?

How many different ways can you find?

Use counters to model each way.

Record each model.

## Show and Share

Share your answers with another pair of students.

How many different ways can the children line up in equal rows?

Write a division fact for each way.

## Connect

There are 12 drum dancers.

They will dance onto the stage in equal rows.

How many dancers could be in each row?

We can use arrays to show the possible ways.



2 rows of 6  
 $12 \div 2 = 6$



3 rows of 4  
 $12 \div 3 = 4$

4 rows of 3  
 $12 \div 4 = 3$

6 rows of 2  
 $12 \div 6 = 2$



1 row of 12  
 $12 \div 1 = 12$

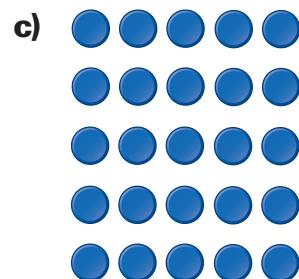
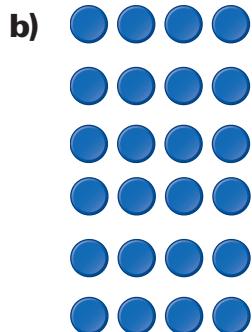
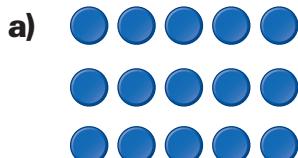


12 rows of 1  
 $12 \div 12 = 1$

## Practice

Use counters when they help.

1. Write a division fact for each array.



**2. a)** Draw an array to show  $5 \div 1$ .  
**b)** Draw an array to show  $5 \div 5$ .

**3. Divide.**  
**a)**  $6 \div 1$       **b)**  $8 \div 1$       **c)**  $7 \div 1$       **d)**  $1 \div 1$   
What patterns do you see?

**4. Copy and complete each division equation.**  
**a)**  $30 \div 6 = \square$     **b)**  $7 \div \square = 7$     **c)**  $\square \div 7 = 4$     **d)**  $36 \div 6 = \square$

**5. The choir sings on stage.**  
**a)** There are 35 chairs in 5 equal rows.  
How many chairs are there in each row?  
**b)** Suppose there are 35 chairs in 7 equal rows.  
How many chairs are there in each row?  
Draw an array to show each answer.

**6. There are 12 drummers and 15 horn players.**  
**a)** Can they form equal rows of 2?  
How do you know?  
**b)** Can they form equal rows of 3?  
How do you know?  
**c)** What other equal rows can they form?  
Show your work.

**7. Explain why the answer is always 1 when you divide a number by itself.**

**8. Write a story problem that you can solve by drawing an array to divide.**  
Solve your problem.  
Show your work.

### Reflect

How can you use an array to divide?  
Use words, numbers, or pictures to explain.





# Relating Multiplication and Division

## Explore

Fifty-six students want to play basketball.  
 There are 7 players on a school team.  
 How many teams can be made?  
 Solve this problem.  
 Use any materials you need.  
 Record your work.



## Show and Share

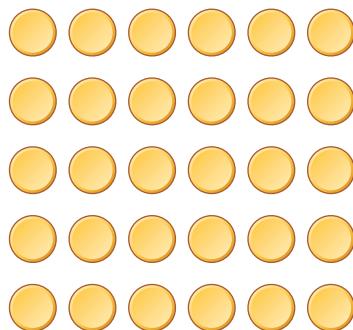
Share your answer with another pair of students.  
 What strategy did you use to solve the problem?

## Connect

Thirty students want to play volleyball.  
 There are 6 players on a team.  
 How many teams can there be?

To find how many teams, divide:  $30 \div 6$

- Make an array of 30 counters with 6 counters in each row.  
 There are 5 rows.



So,  $30 \div 6 = 5$   
 There can be 5 teams.

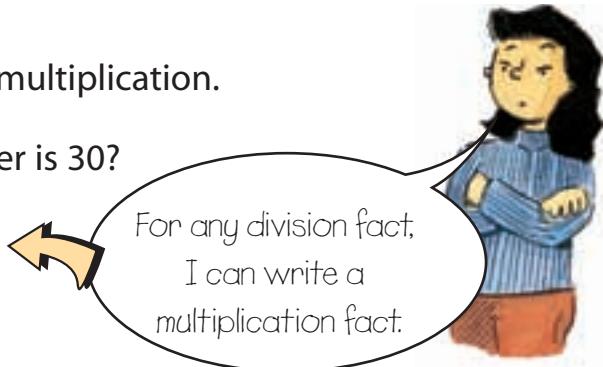
► To find  $30 \div 6$ :

To divide, you can think about multiplication.

Think: 6 times which number is 30?

You know  $6 \times 5 = 30$ .

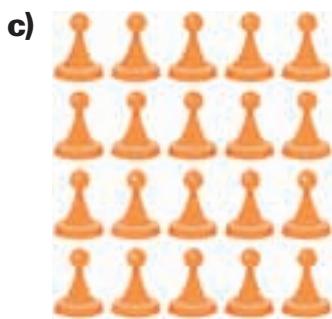
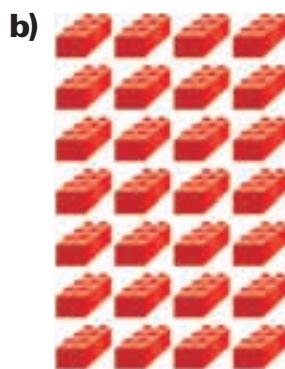
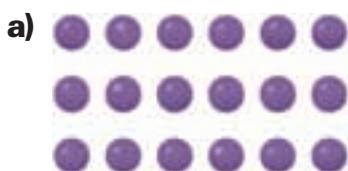
So,  $30 \div 6 = 5$



## Practice

Use counters when they help.

1. Write a multiplication fact and a division fact for each array.



2. a) Draw an array to show  $7 \div 7$ .

b) Draw an array to show  $7 \div 1$ .

3. Use multiplication facts to help you divide.

a)  $6 \times 4 = 24$

b)  $5 \times 7 = 35$

c)  $4 \times 2 = 8$

d)  $9 \times 6 = 54$

$24 \div 4 = \square$

$35 \div 7 = \square$

$8 \div 2 = \square$

$54 \div 6 = \square$

4. Divide. Draw a picture to show your work.

a)  $3 \div 1$

b)  $14 \div 2$

c)  $20 \div 4$

d)  $42 \div 7$

e)  $25 \div 5$

f)  $24 \div 6$

g)  $21 \div 3$

h)  $6 \div 6$

5. Danielle works in a hardware store.

She makes packages of washers.

Danielle puts 6 washers in each package.

She has 36 washers.

How many packages can Danielle make?



6. Use a multiplication fact to divide.

a)  $15 \div 3$

b)  $30 \div 5$

c)  $49 \div 7$

d)  $12 \div 2$

e)  $28 \div 4$

f)  $42 \div 6$

g)  $6 \div 1$

h)  $16 \div 4$

7. Write a story problem you can solve using division.

Trade problems with a classmate.

Solve your classmate's problem.

8. A class made equal teams of 5 for basketball,

and equal teams of 6 for volleyball.

Each student in the class was on a team.

How many students might be in the class?



9. Joe has 35 cubes.

He shares the cubes equally among 7 students.

Each student needs 6 cubes.

Does Joe have enough cubes? Explain.

Show your work.



### Reflect

How can you use what you have learned about multiplication to divide?

Use words, numbers, or pictures to explain.



# Dividing by Numbers from 1 to 9

## Explore



Use the multiplication chart.

- Write the multiplication facts that have 8 as a factor.  
Use these facts to write all the division facts where you divide by 8.  
Draw arrays to show some of these facts.
- Repeat the activity for multiplication facts that have 9 as a factor.

x	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9
2	2	4	6	8	10	12	14	16	18
3	3	6	9	12	15	18	21	24	27
4	4	8	12	16	20	24	28	32	36
5	5	10	15	20	25	30	35	40	45
6	6	12	18	24	30	36	42	48	54
7	7	14	21	28	35	42	49	56	63
8	8	16	24	32	40	48	56	64	72
9	9	18	27	36	45	54	63	72	81

## Show and Share

Share your facts and arrays with another pair of students.

How do you know if you found all the facts?

## Connect

- To find  $72 \div 9$ :

Think  
multiplication:

$$9 \times \square = 72$$

You know:

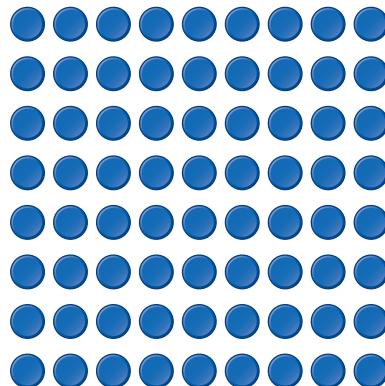
$$9 \times 8 = 72$$

So,

$$72 \div 9 = 8$$

Also,

$$72 \div 8 = 9$$



► To find  $64 \div 8$ :



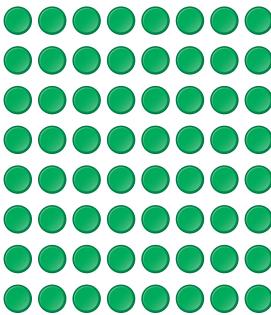
$$8 \times \square = 64$$

You know:

$$8 \times 8 = 64$$

So,

$$64 \div 8 = 8$$



For most multiplication facts,  
you know two division facts.

$$\begin{aligned}7 \times 8 &= 56 \\8 \times 7 &= 56 \\56 \div 8 &= 7 \\56 \div 7 &= 8\end{aligned}$$

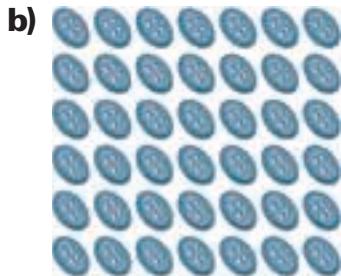
These are **related facts**.

$$\begin{aligned}7 \times 7 &= 49 \\49 \div 7 &= 7\end{aligned}$$

Use counters or grid paper when they help.

## Practice

1. Write two multiplication facts and two division facts for each array.



2. Find each product.

Then write a related multiplication fact and two division facts.

a)  $7 \times 3 = \square$     b)  $8 \times 6 = \square$     c)  $5 \times 9 = \square$     d)  $9 \times 7 = \square$

3. Write four related facts for each set of numbers.

a) 9, 4, 36    b) 5, 8, 40    c) 4, 7, 28    d) 1, 7, 7

4. a) One number in a set of related facts is 63. What could the facts be?

b) One number in a set of related facts is 8. What could the facts be?

5. Divide.

a)  $24 \div 8$     b)  $36 \div 9$     c)  $56 \div 8$     d)  $9 \div 1$     e)  $16 \div 8$   
f)  $72 \div 8$     g)  $63 \div 9$     h)  $27 \div 3$     i)  $8 \div 8$     j)  $64 \div 8$

6. Write all the multiplication facts for which there is only one division fact.

Where are these products on the multiplication chart?

Write each related division fact.

7. Divide.

a)  $35 \div 5$     b)  $45 \div 9$     c)  $81 \div 9$     d)  $18 \div 6$   
e)  $20 \div 4$     f)  $36 \div 9$     g)  $54 \div 9$     h)  $63 \div 9$

8. a) If you know that  $63 \div 9 = 7$ , what else do you know?

b) Write a story problem that could be represented by the equation in part a.

9. Grade 4 students are going on an activities day.

There are 32 students in the class.

Eight students can go in each canoe.

How many canoes will be needed?

Write an equation for this problem.

Solve the equation.



10. There are 9 marbles in each bag.

Heidi wants to buy 54 marbles.

How many bags does Heidi need to buy?

11. Write a story problem that you can solve by dividing.

Trade problems with a classmate. Solve your classmate's problem.

12. a) Is  $48 \div 8$  more or less than  $40 \div 8$ ? How do you know?

b) Is  $72 \div 9$  more or less than  $72 \div 8$ ? How do you know?

Show your work.



### Reflect

How can you use an array to show how multiplication and division are related?

## Pose and Solve Problems

## Explore



Thirty children signed up for sports.

This table shows the sport and the number of players per team.

Sport	Number of Players
baseball	9
basketball	5
soccer	6
frisbee	8

The coaches want every child to be on a team.

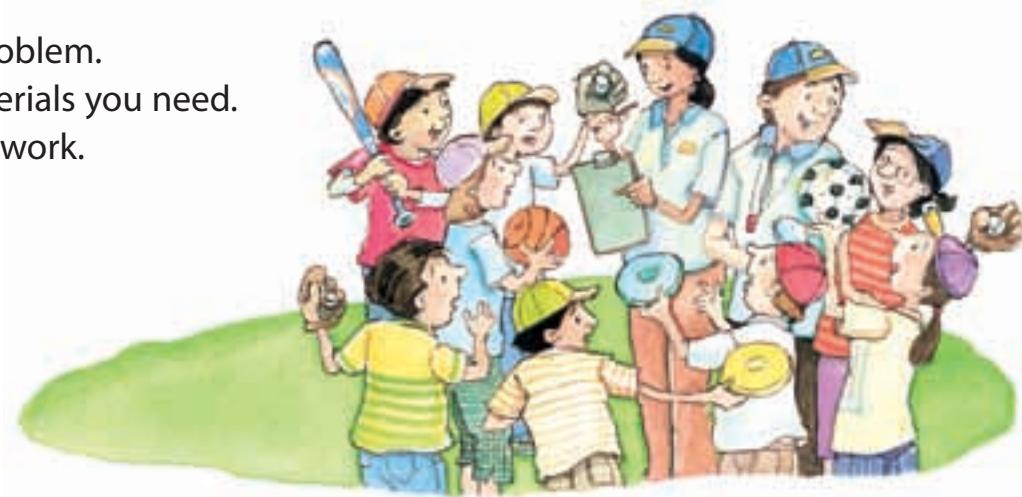
The children play one sport at a time.

Which sports can the coaches choose?

Solve this problem.

Use any materials you need.

Record your work.



## Show and Share

Share your work with another pair of students.

Did you multiply or divide to solve the problem?

What strategy did you use to solve the problem?

## Connect

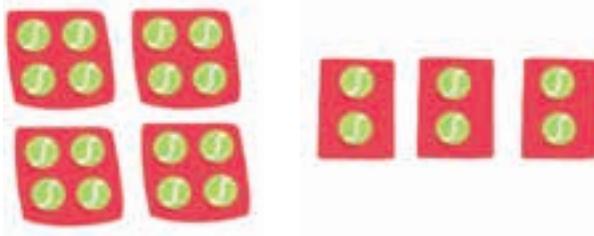
In a tennis match, players play doubles games or singles games.  
4 players are needed for a doubles game.  
2 players are needed for a singles game.  
There are 22 players and 7 games.  
How many doubles games and singles games are there?

Here are two ways to find out.



► Use a model.

Use 22 counters to show the number of players.  
Put the counters in groups of 2 to show singles games.  
Put the counters in groups of 4 to show doubles games.  
Make sure there are 7 groups.



$$\begin{array}{l} 4 \times 4 = 16 \\ \downarrow \qquad \qquad \qquad \downarrow \\ 16 + 6 = 22 \end{array} \qquad \begin{array}{l} 3 \times 2 = 6 \end{array}$$

► Guess, then test.

Suppose you guess 2 doubles games and 5 singles games.  
 $2 \times 4 = 8$ ; that is 8 players playing doubles.  
 $5 \times 2 = 10$ ; that is 10 players playing singles.  
Test:  $8 + 10 = 18$ ; that is too few players.

Try another guess.

Guess 4 doubles games and 3 singles games.  
 $4 \times 4 = 16$ ; that is 16 players playing doubles.  
 $3 \times 2 = 6$ ; that is 6 players playing singles.  
Test:  $16 + 6 = 22$ ; yes, 22 is the correct number of players.

There are 4 doubles games and 3 singles games.

## Practice

Use counters if they help. Show your work.

1. Forty-two students want to play volleyball.  
There are 6 players on a team.  
How many teams can there be?
2. There should be a water bottle for each player in the tournament.  
There are 8 teams. Each team has 6 players.  
How many water bottles are needed?
3. Write a story problem for each situation. Then solve the problem.
  - a) There are 24 water bottles for 4 teams.
  - b) There are 8 teams with 9 players on each team.
4. The coach has between 40 and 50 ribbons for the track meet.  
She has an equal number of ribbons for each of the 6 events.  
There are 5 ribbons left over.  
How many ribbons might there be for each event?
5. Thirty-five students signed up to play hockey.  
There can be 7 teams in the tournament.  
Each team should have 6 players.  
Are there enough players to make 7 teams? How do you know?
6. Use the data in the table.  
Write 3 story problems you can solve using multiplication or division.  
Solve each problem.
7. Write a story problem you could solve by finding  $9 \times 5$ .

Sport	Players on a Team
Baseball	9
Basketball	5
Ice hockey	6

## Reflect

How do you choose a strategy to solve a story problem?  
Which strategy do you use often? Explain.

## LESSON

Use any materials when they help.

1. Multiply. What strategies did you use?

a)  $3 \times 2$       b)  $3 \times 4$       c)  $3 \times 8$   
 d)  $2 \times 6$       e)  $4 \times 6$       f)  $8 \times 6$

2. Multiply. What strategies did you use?

a)  $5 \times 8$       b)  $1 \times 8$       c)  $9 \times 0$       d)  $7 \times 6$   
 e)  $9 \times 9$       f)  $8 \times 10$       g)  $8 \times 9$       h)  $4 \times 8$

3. There are 8 cereal bars in a box.

a) How many bars are in 2 boxes?  
 b) How many bars are in 4 boxes?  
 c) How many bars are in 8 boxes?

How could you use one answer to get the next?



4. Copy this chart. Fill in the missing numbers.

Describe each skip counting pattern.

$\times$	4	5	6	7	8	9
4	16	20				
5		25		35		
6			36			54

5. Ali knows that  $7 \times 6 = 42$ .

How can he use this fact to find  $7 \times 7$  and  $7 \times 8$ ?

6. The answer to a multiplication question is 24.

What might the question be?

Write as many multiplication facts as you can.

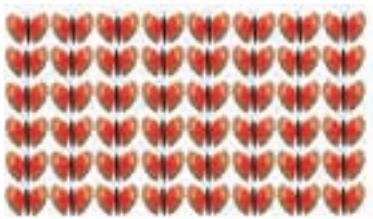
7. Use words, numbers, or pictures to explain your thinking.

a) How might you use  $4 \times 7$  to find  $8 \times 7$ ?  
 b) How might you use  $5 \times 5$  to find  $5 \times 6$ ?  
 c) How might you use  $3 \times 10$  to find  $3 \times 9$ ?

7

8. Write a multiplication fact and a division fact for each array.

a)



b)

5  
8

9. What might each missing number be? How many answers can you find?

a)  $\square \times \circ = 35$    b)  $\triangle \div \square = 1$    c)  $\square \times \circ = 63$    d)  $\square \div \triangle = 5$

10. Divide.

a)  $45 \div 9$    b)  $32 \div 8$    c)  $56 \div 7$    d)  $27 \div 3$    e)  $9 \div 9$

9

11. Find each product.

Then write a related multiplication fact and two division facts.

a)  $6 \times 7 = \square$    b)  $6 \times 9 = \square$    c)  $8 \times 3 = \square$    d)  $5 \times 7 = \square$

12. Write four related facts for each set of numbers.

a) 9, 7, 63   b) 4, 5, 20   c) 6, 8, 48

13. Which multiplication facts will help you find the answers?

a)  $30 \div 6$    b)  $64 \div 8$    c)  $42 \div 7$

14. There are 8 flowers in a bunch.

Suppose you want to buy 40 flowers.

How many bunches would you need to buy?

Write an equation for the problem.

Solve the equation.

10

15. Use the data below.

Write a problem that can be solved by multiplying or dividing.

Solve your problem.

There are 48 apples.

There are 5 boys and 3 girls.



## Learning Goals

- use different mental math strategies to multiply and divide
- multiply by 0, 1, and 10
- divide by 1
- recall multiplication and division facts
- identify and describe patterns in a multiplication chart
- relate multiplication and division
- pose and solve story problems using multiplication and division
- write and solve equations

## Unit Problem

# Here Comes the Band!



### Part 1

A marching band has 48 people.

They march in equal rows.

How many different ways can the band be arranged?

Write a multiplication fact for each way.

Show your work.

## Check List

Your work should show

- all the ways to arrange the band members in equal rows
- correct multiplication and division facts
- a clear explanation of why 31 band members might be a problem
- how you used mathematical language and symbols correctly

### Part 2

- Thirty band members perform onstage. You will set up chairs for them.
  - There must be equal rows of chairs.
  - There must be at least 2 chairs in each row.How many ways can you set up the chairs? Write an equation you can solve to find each way.
- What problems might you have if the band has 31 members? Explain.

### Part 3

Suppose you are the bandleader for a day.

- You choose how many band members will play that day.
- Choose a number.
- Show all the different ways you could arrange your band members into equal rows.

Write a multiplication fact and a division fact for each way.



### Reflect on Your Learning

How are multiplication and division related?  
Use words, pictures, or numbers to explain.

## UNIT

1

- What patterns do you see in each set of numbers?
  - the blue numbers
  - the red numbers
  - the green numbers
- Use the multiplication chart in question 1.  
Find 3 patterns different from those in question 1.  
Write a rule for each pattern.

$\times$	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	10	12	14	16	18
3	0	3	6	9	12	15	18	21	24	27
4	0	4	8	12	16	20	24	28	32	36
5	0	5	10	15	20	25	30	35	40	45
6	0	6	12	18	24	30	36	42	48	54
7	0	7	14	21	28	35	42	49	56	63
8	0	8	16	24	32	40	48	56	64	72
9	0	9	18	27	36	45	54	63	72	81

- Find the missing numbers in this addition chart.  
Explain how you could use patterns to find these numbers.

+	4	8	12	16	20
2	6		14	18	
4	8	12		20	24
6	10		18		26
8	12	16		24	
10		18		26	30

- Find the errors in this multiplication chart.  
How did you find each error?

$\times$	10	8	6	4	2
2	20	16	12	8	6
4	40	30	24	16	8
6	60	48	36	26	12
8	80	64	50	32	16
10	100	80	60	40	20

2

5. Choose a 4-digit number. Write it in:

a) standard form    b) expanded form  
 c) words                    d) a place-value chart

6. Use the digits 3, 4, 5, 6.

a) Write all the 4-digit numbers greater than 4000 and less than 5000.  
 b) Order the numbers from greatest to least.  
 Use place value to explain how you did this.

7. Estimate first. Then add or subtract the numbers for which the answer is greater than 500.

a) 
$$\begin{array}{r} 219 \\ - 195 \\ \hline \end{array}$$

b) 
$$\begin{array}{r} 627 \\ + 186 \\ \hline \end{array}$$

c) 
$$\begin{array}{r} 87 \\ + 256 \\ \hline \end{array}$$

d) 
$$\begin{array}{r} 786 \\ - 195 \\ \hline \end{array}$$

3

8. Multiply. Use mental math.

Try to use a different strategy each time.

a)  $6 \times 3$     b)  $6 \times 6$     c)  $6 \times 0$     d)  $2 \times 7$     e)  $4 \times 7$   
 f)  $5 \times 7$     g)  $7 \times 7$     h)  $8 \times 9$     i)  $4 \times 1$     j)  $5 \times 9$

9. Megan has 8 boxes of books.

Each box contains 8 books.

How many books does Megan have altogether?

10. Divide. How can you use multiplication to help you?

a)  $20 \div 4$     b)  $42 \div 7$     c)  $5 \div 5$     d)  $27 \div 3$     e)  $16 \div 2$   
 f)  $48 \div 8$     g)  $30 \div 6$     h)  $81 \div 9$     i)  $21 \div 7$     j)  $32 \div 4$

11. Choose one part of question 10.

Draw an array to show how to divide.

12. Ali has 56 apples to share equally among 7 people.

How many apples does each person get?

How do you know your answer is correct?